Quasi-Optical Grill Launching of Lower-Hybrid Waves for a Linearly Increasing Plasma Density

Riccardo Borghi, Fabrizio Frezza, Senior Member, IEEE, Giorgio Gerosa, Life Associate Member, IEEE, Massimo Santarsiero, Carlo Santini, and Giuseppe Schettini, Member, IEEE

Abstract—An analysis of quasi-optical grills for lower-hybrid waves for heating and current drive purposes is presented. The tokamak plasma density versus the abscissa entering the plasma is assumed to behave like a step and a subsequent ramp. The study is performed by means of a two-dimensional formulation employing cylindrical waves. A detailed numerical analysis is presented, which allows us to show results for different configurations useful in practical cases and comparisons with the constant-density case.

Index Terms—Cylinders, cylindrical arrays, electromagnetic coupling, electromagnetic launching, electromagnetic scattering, plasma heating, plasma waves.

I. INTRODUCTION

A s is well known, mechanisms currently used to couple radiofrequency radiation to lower-hybrid plasma waves for heating and current-drive purposes make essential use of evanescent waves [1]. Waveguide grills [2] are presently the most widely used devices in toroidal plasmas, due to the very high flexibility regarding both the launched spectrum and the antenna directivity. However, thousands of waveguides are needed for next-step devices (like International Thermonuclear Experimental Reactor) with obvious handling problems.

In recent years, many alternative solutions have been proposed and developed to simplify the layout and the operation of the coupling structure. Among them, we recall the multi-junction [3], which subsequently gave rise to the hyperguide [4], and the quasi-optical grills [5]. In the latter structures the excitation of the plasma lower-hybrid wave is produced by means of the scattering of a radiofrequency beam by a grating of conducting rods, with a drastic reduction in dissipation and complexity of the coupling structure.

The analysis of the coupling of a plane wave, propagating in a vacuum, with a lower hybrid plasma wave by means of a quasi-optical grill, for nuclear fusion purposes, has been tackled with different techniques [3], [5]–[7], giving results in agreement especially for single-line gratings.

In a previous paper [6] we studied the scattering problem of a plane wave by a set of perfectly conducting circular cylinders placed close to a plasma interface. We presented a full-wave solution and gave numerical results for the case of a constant-density plasma, which is simpler from a numerical point of view [8], [9], though important to have estimates of the coupling parameters. In this paper, we intend to extend the results presented there to the case of a linearly-increasing plasma density, which represents a more realistic model [9], [10].

The general lines of the method, briefly recalled in Section II, closely follow those of [6] and [11]. The main novelty presented here, i.e., the treatment of a linearly-increasing plasma density, is made possible by the development of fast and accurate integration methods for the highly oscillating functions involved in the problem. In Section III numerical results are presented, compared with those obtained for the case of a constant-density plasma, while future developments and conclusions are briefly discussed in the last section.

II. THEORETICAL ANALYSIS

A. Solution of the Scattering Problem

The problem under investigation is the scattering of an electromagnetic, linearly polarized plane wave, with wavevector \( \mathbf{k} \), impinging on a group of perfectly conducting parallel cylinders placed near a plane plasma surface, parallel to the cylinders (see Fig. 1). \( \varphi \) is the angle between the wavevector \( \mathbf{k} \) and the direction perpendicular to the surface,
Fig. 9. Reflected power (%) versus normalized density \( n_0/n_c \); \( N = 10 \), \( k_{0d} = 0.85 \), \( k_{0d} = 2.9 \), \( k_{0d} = 0.25 \), \( \varphi = 45^\circ \), and \( f = 10 \) GHz.

Fig. 10. Directivity (%) versus \( n_0/n_c \). The other parameters are the same as in Fig. 9.

Fig. 11. Array of pairs of cylinders.

A more efficient configuration has been proposed in [18] and [19] for a Gaussian beam illumination and a constant-density plasma. A consistent analysis for that configuration and for a plasma density of the kind considered in this paper can be performed once an incident Gaussian beam is assumed. This problem is an extension of the present method and its analysis is in progress.

IV. CONCLUSIONS

In this paper, we have shown the effects of a quite realistic model for the external layer of a thermonuclear plasma on the coupling parameters (reflected power and directivity) when a quasi-optical grill of conducting circular cylinders is used to launch lower-hybrid slow plasma waves. Such a model considers that the electron density, versus the distance from the plasma surface, behaves like a step followed by a linear ramp.

The analytical and computational difficulties arisen, with respect to the constant-density case, have been discussed and overcome. Numerical results have been presented for the coupling parameters for various geometrical configurations and physical parameters. Such results have shown to tend to the ones pertinent to the constant-density case when the slope of the ramp tends to zero.

As a general remark, we have found that the performances of the coupler become systematically worse on increasing the plasma density slope, so that the values of the coupled power and directivity are generally lower than those evaluated by modeling the plasma density as a constant.

The analysis could be extended to the case of an incident Gaussian beam, representing a more realistic model for the incident field. This could be performed by means of a suitable plane-wave representation of the Gaussian beam, following the lines of [18], [19]. Furthermore, the more efficient configurations proposed in the latter references could be profitably analyzed for the case of a plasma with linearly increasing density, when an incident Gaussian beam is considered. Finally better performances may be obtained using a systematic optimization procedure or employing scattering elements of noncircular cross sections, such as elliptical [9].
or rectangular [20], [21] ones. The limits of applicability of our method do not differ much from the ones given in [2] where a similar problem was faced for a waveguide grill launcher. In particular, the effect of the plasma curvature should be taken into account, adapting the presented model to the various tokamaks making use of suitable approximations. Our approach can be easily extended to the case of an obliquely incident plane wave with respect to the cylinder axes, and this is the key to generalize the technique to the case of finite height cylinders by means of a suitable Fourier expansion (with respect to the y-direction).

Finally, as is well known, the plasma density undergoes random fluctuations which are usually neglected. A future goal of our work is to include these effects using statistical methods.

**Appendix**

**Numerical Computation of \( \Gamma(\eta) \)**

The computation of the Airy functions for values of \( k_0 \Delta \) greater than \( 10^{10} \text{ m}^{-1} \) can be performed by using their relation with the Bessel functions of fractional order or, alternatively, their expansion in ascending series [16]. These techniques are very efficient and their implementation does not present numerical difficulties.

For smaller values of \( k_0 \Delta \), the asymptotic expansions of \( Ai \), \( Ai' \), \( Bi \), and \( Bi' \) may be used [16]. However, such expansions give rise to loss of precision in the limit \( \Delta \to 0 \). Nevertheless, it is important to verify that in such limit the linear density plasma model results match those obtained for the constant density one. To this aim, it is possible to obtain different forms for (35) and (44), which allow an easy and fast computation of the \( \Gamma(\eta) \) function for very small values of \( k_0 \Delta \). In particular, we note that

\[
\lim_{\Delta \to 0} u_{\eta_0} = +\infty.
\]

Thus, the asymptotic form for \( \Gamma(\eta) \) with \( \Delta \) approaching zero may be obtained from the expansion of (35) and (44) for large values of \( u_{\eta_0} \). It is useful, therefore, to express (35) and (44) as functions of \( u_{\eta_0} \) only. From (27), (34), and (38), in both cases \( |\eta| < 1 \) and \( |\eta| > 1 \), we get

\[
\left( \frac{\Delta}{\eta_0} \right)^{1/3} = \left( \frac{1}{u_{\eta_0}} - 1 \right) \left( 1 - |\eta_0| \right)^{1/6}.
\]

Substituting (47) into (35) and (44), we obtain

\[
\eta(\eta_0) = \left\{ \begin{array}{ll}
\frac{1}{\sqrt{u_{\eta_0}} \eta_0} \left[ (\eta_0 - \eta_0^{-1}) \right]^{1/2} & |\eta| < 1 \\
\frac{1}{\sqrt{u_{\eta_0}} \eta_0} \left[ (\eta_0 + \eta_0^{-1}) \right]^{1/2} & |\eta| > 1.
\end{array} \right.
\]

By using the asymptotic expansions of the Airy functions [16] for large arguments, (48) in the limit \( \Delta \to 0 \), turns into (49) shown at the top of the page where

\[
z_0 = \frac{2}{3}\eta_0^{3/2}
\]

and

\[
\begin{align*}
\alpha_0 &= 1 \\
\alpha_k &= \frac{2(k+1)(2k+3)(6k+1)}{2(k+1)!} \\
\beta_0 &= 1 \\
\beta_k &= \frac{(6k+1)}{(2k+1)!} (k = 1, 2, \cdots).
\end{align*}
\]

By substituting the \( \eta \) values of (49) into (20) the expressions of the reflection coefficient \( \Gamma(\eta) \) are easily obtained. Numerical tests have proved that this procedure is very efficient, accurate, and reliable in a wide range of \( \Delta \) values.

**References**


Dr. Frezza is a member of Sigma Xi, the Electrical and Electronic Italian Association (AEI), and of the Italian Society of Applied and Industrial Mathematics (SIMAI).

Massimo Santarsiero received the Laurea degree in physics in 1988 and Ph.D. degree in applied electromagnetics in 1992 from “La Sapienza” University of Rome, Italy.

He worked at the Italian Energy and Environment Agency (ENEA) on holography and interferometry. Presently he is at the Department of Physics of the University “Roma Tre,” Rome, Italy, where he works on optical coherence, laser beam characterization, electromagnetic scattering by microstructures.

Dr. Schettini is a member of the Italian Physical Society (SIF), the Italian Optics and Photonics Society (SIOF), “La Sapienza” unit of the Electromagnetics Group of Italian National Research Council (CNR), and “La Sapienza” unit of the Electronics CNR Group.

Massimo Santarsiero received the Laurea degree in electronic engineering in 1996 from “La Sapienza” University of Rome, Italy.

He presently collaborates with the Department of Electronic Engineering of “La Sapienza” University of Rome on electromagnetic scattering problems.

Giuseppe Schettini (S’84–M’96) received the Laurea degree in electronic engineering in 1986, the Ph.D. degree in applied electromagnetics and electrophysics sciences in 1991, and the Laurea degree in physics in 1995, all from “La Sapienza” University of Rome, Italy.

After his graduation in electronic engineering, he joined the Italian Energy and Environment Agency (ENEA) where he first worked on free electron lasers and then on the radiofrequency heating of thermonuclear plasmas. In 1992 he joined “La Sapienza” University of Rome as a Researcher and Assistant Professor of Electromagnetics. Since 1995 he has been a temporary Professor of Electromagnetic fields. His scientific activity is focused on radiofrequency heating of thermonuclear plasmas, ferrite resonators, propagation and diffraction of optical beams, diffractive optics, and free electron lasers.

Dr. Schettini is a member of the Italian Physical Society (SIF), the Italian Optics and Photonics Society (SIOF), “La Sapienza” unit of the Electromagnetics Group of Italian National Research Council (CNR), and “La Sapienza” unit of the Electronics CNR Group.